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REVIEWS

Mean-Field Magnetohydrodynamics and Dynamo Theory. By F. KRAUSE AND K.-H. RÄDLER. Pergamon, Oxford OX3 0BW 1980. 271 pp. £15.

Mean-field magnetohydrodynamics (MFM) is one of the techniques developed during attempts to explain why magnetic fields are almost certain to be generated by irregular motions in a large rotating mass of electrically conducting fluid like the Sun or the Earth's core. MFM was initiated about the year 1965 by M. Steenbeck, F. Krause and K.-H. Rädler. It assumes that the fluid motions and the magnetic field can each be divided into a large-scale slowly varying mean part and a small-scale, rapidly varying turbulent part. Interaction between the turbulent motion and field can yield a mean EMF $\mathscr{E}$ with a non-vanishing component $\alpha\bar{\mathbf{B}}$ in the direction of the mean magnetic field $\bar{\mathbf{B}}$: this is the α-effect, vital if the body is to function like a self-excited dynamo. In 1971, NCAR published an English translation (by P. H. Roberts and M. Stix) of some 14 papers on MFM up to that date. The volume at present under review can be regarded as codifying the papers in the NCAR collection and bringing it up to date.

The volume is mainly mathematical, though not deeply so. To ensure that its ideas are adequately understood, expositions are given of certain purely mathematical topics, e.g. isotropic tensors and different modes of averaging. The steps in mathematical arguments are explained meticulously (perhaps even excessively so: it is sometimes hard to see the wood for the trees). About half of the book is concerned with turbulence and the calculation of $\mathscr{E}$. The expression for $\mathscr{E}$ is found to be far more complicated than a simple $\alpha\bar{\mathbf{B}}$, and to involve a number of parameters whose magnitudes can at best be estimated only roughly: however, in applications most of the complications are smoothed away.

The applications, as stated above, refer to attempts to explain the magnetic field of bodies like the Earth or Sun in terms of a dynamo. In them, attention is usually confined to kinematic dynamos, i.e. those in which the motion is taken as given and unaffected by growth of the magnetic field. The back-reaction of the field on the motion is indeed briefly considered, but its effect in limiting the growth of the magnetic field is represented only by an *ad hoc* assumption about a dependence of α on $\bar{\mathbf{B}}$. I felt least happy about the applications section. In the early 1970s, when some doubt still persisted about the efficacy of α-effect dynamos, a large number of computations were made which removed most of these doubts. The computations assumed rather artificial distributions of α and the angular velocity ω throughout the fluid mass, distributions chosen for their mathematical simplicity rather than for physical reasons. They achieved their main purpose in showing that dynamo maintenance of the magnetic field was explicable in terms of MFM, but some of the conclusions drawn from them were later shown to be incorrect when more-realistic distributions of α and ω were assumed. I felt that the book paid too much attention to these early computations. The account of the observed properties of the magnetic fields which it was desired to explain also seemed rather inadequate.

None the less, it is well to have this authoritative account of MFM and its importance to dynamo theory. Considering that it was produced in English by German authors, the book contains remarkably few errors. It has a very good bibliography, which will be useful to readers wanting to know more about other dynamo theories, hinted at but not discussed in detail here.

T. G. COWLING

Mechanics of Swimming and Flying. By STEPHEN CHILDRESS. Cambridge University Press, 1981. 155 pp. £17.50 (hardback), £7.50 (paperback).

A knowledge of fluid dynamics is central to an understanding of animal locomotion, so it is not surprising that when the application of physics and mathematics to biology began to flourish, in the late 1960s and early 1970s, there was a burst of fluid-dynamical activity in this field. The work was dominated by Lighthill, who published surveys of high-Reynolds-number swimming (1969, 1970, 1971), low-Reynolds-number swimming (1975) and animal flight (1974). These surveys were gathered together and extended in his book *Mathematical Biofluiddynamics*, published by SIAM in 1975, which together with his 1976 review 'Flagellar hydrodynamics' (*SIAM Rev.* vol. 18, p. 161) and his 1977 'Introduction to the scaling of aerial locomotion' (in *Scale Effects in Animal Locomotion*, Academic) constitute a fairly complete general picture of the standard modes of locomotion in fluids. 1975 was also the year in which the two volume conference proceedings *Swimming and Flying in Nature* (SF) edited by T. Y. Wu, appeared, containing many papers on the details of particular modes of locomotion. Since that time the general has increasingly given way to the particular, and the fluid dynamics has become increasingly difficult, as biologists and fluid dynamics have sought to elucidate the biological role of particular 'design' characteristics (see e.g. Rayner *J. Exp. Biol.* vol. 80, 1979, p. 17; Higdon *J. Fluid Mech.* vol. 94, 1979, pp. 305 and 331).

The present monograph (one of a new series of 'Cambridge Studies in Mathematical Biology') arose out of a course of lectures given at the Courant Institute in 1976 with the aim of providing a brief complement to Lighthill's book and to SF, and of contributing to the general literature, not the particular. The author intends it to be accessible to 'upper-level undergraduates and first year graduate students with a firm understanding of introductory fluid mechanics', and to provide for them a 'compact introduction to biological modelling'. There is no doubt that he has succeeded in all these aims; in particular, I am sure that the lecture course did provide a most useful complement to Lighthill's book and reviews, given that the students were reading them at the same time. Childress gives the mathematical derivation of a number of results for which Lighthill argued on more physical grounds, and the combination of the two approaches leads to a truly deep understanding (although the increase of mathematical formalism does not in all cases add to the rigour of the argument, e.g. in the statement about uniform validity of a low-Re result as $Re \to \infty$, p. 30).

However one might ask if those are sufficient grounds for writing the lectures up into a book? I still believe that the best book for a lecture course is the student's own notebook, and that if one is going to recommend a textbook, it should add to the lectures, not merely repeat them. In the case of animal locomotion one would want fluid-dynamics students to have access both to a suitable biology text, to see the sort of questions a biologist might ask, and on the other hand to a definitive fluid-dynamical treatise in which the subject is dealt with comprehensively, or at least more completely than in the lectures. This book makes no claim to fulfil either of these roles.

There are twelve chapters: one is an introduction, six are on low-Reynolds-number swimming, three on high-Reynolds-number swimming, only one on flight, and the final one is a collection of various topics, including the author's own contribution, the biologically somewhat obscure but mathematically highly entertaining subject of bioconvection. Almost the only biology is an extremely abbreviated discussion of microorganisms, placed for some reason in chapter 4. Of the high-Reynolds-number chapters, chapter 9 and most of chapter 8 contain the introductory fluid mechanics with which the reader is supposed to be already familiar. It can thus be seen that the promise of the title is fulfilled only very unevenly. Three pieces of quite difficult theory are covered in some depth: the analysis of a single flagellum (chapter 6), slender-body theory for fish swimming (chapter 10) and unsteady lifting-line theory (chapter 11), but even in these cases the implications are scarcely followed up.

On the whole the author displays his mathematics clearly, but occasionally, especially in physical deduction, he is terse to the point of obscurity, as if he prefers to make the student struggle for understanding than to help him with a clear, if longer, explanation. Examples include the discussion of time-reversal symmetry on p. 17, the implications of non-zero-thrust swimming on p. 45 (where the phrase 'in other words' precedes a complete non-sequitur), and an application of Kelvin's theorem to vortex shedding on p. 105. His clarity is not helped by a number of misprints, of which the most important appears on p. 44, during the statement of the 'fundamental assumption of resistive force theory' where a force is equated to a scalar and on the next line to a second-rank tensor (the conclusion, equation (5.6), is correct). Others include muddled captions to figures 3.1 and 3.2, a line (or more?) missing on p. 113, and Lighthill's initials given incorrectly in the list of references.

In conclusion, this is not an altogether satisfactory book, although if I were giving a course on animal locomotion I would find it very useful. However I would not recommend it to the students until I was sure they had already read the key points in Lighthill's own words.

T. J. PEDLEY

The Inverse Scattering Transformation and the Theory of Solitons. By W. ECKHAUS AND A. VAN HARTEN. North-Holland, 1981. 222 pp. $31.75.

An evocative terminology is a catalyst for promoting the growth of a subject area. Zabusky & Kruskal (*Phys. Rev. Lett.* vol. 15, 1965, pp. 240–243) provided this vital ingredient when they coined the word 'soliton' to describe solitary waves which have the additional strong stability property of preserving their identity even under spectacularly nonlinear interactions with other solitons. Now, nearly two decades on, the subject of solitons is sufficiently developed to warrant a veritable flood of books written from different perspectives.

Eckhaus & van Harten have written a rigorous mathematical book intended to counterbalance a literature that is strongly orientated towards physical applications. One measure of the level of abstraction is that, in a subject rich with visually attractive phenomena and mathematical solutions, there is but a single diagram. This diagram does concern the interaction between two solitons. However, instead of showing the drama of the apparent destruction and miraculous reconstitution of the soliton structure, the diagram shows a spider's web of straight lines indicating the decay exponents in the numerator and denominator of a mathematical expression.

Also, the diagram with its handwritten equations shows at its worst the disadvantages of a photocopied typescript.

The first chapter is a very readable account of the history and mathematical properties of the Korteweg–de Vries equation. I was particularly impressed with the nice motivation for the Miura transform to the linear Schrödinger equation. The second chapter comes as a bit of a jolt. The reader is thrown right into the inverse-scattering transform with 'a considerable amount of hard explicit computations and estimates'. The third chapter with its neat analysis and punning sub-title – The Lax Approach – comes as light relief. However, with chapter 4 it is back to the scattering problem. After this the generalizations to matrix systems and to non-uniformities in the remaining thıee chapters are comparatively straightforward. A perennial blessing throughout the book is that, although a great deal of concentration is given to make each step totally secure, there are regular reminders and explanations of the overall plan.

This is certainly not a book to be dipped into or skimmed. There is a lot of notation to be absorbed in order to follow the arguments, and many details are left to the reader. However, for any mathematician interested in solitons the effort of working through Eckhaus & van Harten's book is well worth it.

RONALD SMITH

CORRIGENDUM

The response of a turbulent boundary layer to lateral divergence

By A. J. SMITS, J. A. EATON AND P. BRADSHAW

Journal of Fluid Mechanics, vol. 94, 1979, pp. 243–268

The vertical scales of figures 9 and 16 are shown too small by a factor of two. For figure 9 the vertical scale should span from zero to 0.006 instead of zero to 0.003. For figure 16 the vertical scale should span from zero to 0.8 instead of zero to 0.4.

In addition, figure 21 indicates station numbers, not s, the distance along the surface. To find the corresponding s, consult table 7.